Euler reciprocal Relation

let $Z \rightarrow$ State function of two independent variables x and y of the system.

i.e, $Z = f(x, y)$

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$= M(x, y)\,dx + N(x, y)\,dy$$

Taking mixed second derivatives,

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial y \partial x} \quad \text{and} \quad \left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial x \partial y}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

$$\boxed{\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y}$$

Euler reciprocal relation (Reciprocity)

Applicable to state functions only.

$$\oint dz = 0$$

The cyclic rule. When change occurs at constant z (8)

$dz = 0.$

$$0 = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy.$$

$$\left(\frac{\partial x}{\partial y}\right)_z = -\left(\partial z/\partial y\right)_x \Big/ \left(\partial z/\partial x\right)_y$$

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = -1$$

It is called cyclic rule. → applicable for state functions only